

Impact of Product Demand Uncertainties on the Optimal Design of a Sustainable Dairy Supply Chain: A Case Study of Bulgaria

Elisaveta G. Kirilova^{a,*}, Natasha Gr. Vaklieva-Bancheva^a, Rayka K. Vladova^a, Tatyana S. Petrova^a, Desislava S. Nikolova^b, Evgeniy Iv. Ganeev^a, Yunzile R. Dzhelil^a

^aBulgarian Academy of Sciences, Institute of Chemical Engineering, Akad. G. Bontchev Str. Bl. 103, 1113 Sofia, Bulgaria

^bProf. d-r. Asen Zlatarov University, No.1 "P

of. Yakimov" Bul., 8010 Burgas, Bulgaria

e.kirilova@iche.bas.bg

Population growth and income, together with urbanization, have caused a significant increase in demand for dairy products. This creates opportunities for increasing the profit from dairy production, but on the other hand, it is associated with the generation of large amounts of pollutants that are released into the air and water and require costs for their treatment and disposal. The presence of fluctuations in the product demands in the markets also influences the sustainable operation of considered supply chain (SC) activities. This study proposes a robust optimisation approach for handling the uncertainty of product demands in a dairy SC to produce different dairy products according to different recipes while satisfying environmental and economic criteria. The latter is associated with the generated wastewater from dairy production and CO₂ emissions due to the energy consumed and transportation. The approach has been implemented in a real case study from Bulgaria. Deterministic and robust optimization problems have been formulated and solved under nominal data for the product demands and three different uncertainties levels – 0.2, 0.5 and 1. The obtained results show that the increase in the uncertainty level leads to decreasing profit from the dairy SC with a relatively small standard deviation. The lowest mean value of the SC profit of 232,882 BGN is obtained at the greatest uncertainty level of 1. The results for SC total costs show that they also do not change significantly with an increase in the uncertainty level. The largest value of 154,018 BGN has been obtained at an uncertainty level of 0.5. Given the latter, it can be said that the developed robust optimization model is a sustainable, which leads to obtaining results for the SC profit and costs that do not change significantly with an increase in the uncertainty level of consideration of product demands.

1. Introduction

Given the trend toward ever cleaner dairy production, as well as the number of challenges facing this sector, it has necessitated the application of the strategy of sustainable management by optimizing the activities in the SCs. Against this background, the integration of uncertain aspects is constantly becoming important for the decision-making process, since it can result in increased efficiency and market competitiveness in the dairy sector. Over the last decades, the quantitative modelling approaches have found wide application in the sustainable management of the uncertainties that may arise in the operation of real dairy SCs. They include robust optimisation models for handling the uncertainties in closed-loop SC networks problems while satisfying either economic (Pishvae et al., 2011) or simultaneously economic and environmental criteria (Yavari and Geraeli, 2019) have proposed. They represent an extension of deterministic mixed-integer linear programming (MILP) models with a robust counterpart for the uncertain products demands, returns, transportation costs and the quality of returned products. Yang et al. (2015) have proposed a two-stage fuzzy optimization method for dairy SC design problems with uncertain transportation costs and product demands where the model decision variables are defined at different stages. The decision variables related to activated plants and warehouse location have been defined at the first stage, while at the second stage, the quantities of flows along with the

network are determined after knowing the values of costs and demands. Stefansdottir et al. (2018) have proposed a two-stage stochastic MILP model that integrates the selection of new product designs and processing technologies in a dairy SC context. In this approach as uncertain parameters, the product demands with regard to product specifications and volumes have been considered. Gao and You (2018) have proposed a stochastic bi-level MINLP model for optimization of decentralized dairy SCs considering multiple stakeholders under uncertainty. Dutta and Shrivastava (2020) have developed a scenario-based approach for modelling the disruptions in transportation routes within the facilities in milk product SC operating under products demand, supply and process uncertainties. A newsvendor model has been incorporated into the last echelon of SC to handle the retailer-level uncertainty due to demand-supply mismatch. Jouzdani and Govindan (2021) have developed a multi-objective mathematical programming model to optimize the total costs, energy consumption and the traffic congestion associated with dairy SC operations where the three aspects of sustainability have been considered. The authors have considered the product lifetime as an uncertain parameter. Recently, Kirilova et al. (2022) have proposed a deterministic approach for optimal design of sustainable dairy SC for the production of different types of dairy products according different recipes, while satisfying economic, environmental and social criteria defined in terms of costs. The environmental criterion includes assessments of pollutant emissions in relation to two areas of impact - air and water. The latter are associated with CO₂ emissions generated due to transportation and energy consumed and wastewater from the production of dairy products.

It can be concluded from the presented above literature view that the developed approaches for the optimal design of dairy SCs are based on robust, fuzzy or stochastic optimization MILP or MINLP models while satisfying multiple objectives such as minimum total costs, minimum greenhouse gas emissions associated with the transportation and energy consumption, etc. There is no optimisation approach for the design of dairy SC operating under uncertainties that would simultaneously account for the environmental impact of pollutants released in different areas.

The present study represents an extended version of the deterministic approach of Kirilova et al. (2022) where the deterministic model for the optimal design of sustainable dairy SC has been extended with a Robust Counterpart (RC) for the uncertain products demands using the approach of Ben-Tal et al. (2005).

2. Problem definition

A robust optimization model for the design of three-echelon dairy SC operating under uncertain product demands is considered. The SC includes suppliers s , dairies i and markets m . Different products p should be produced in the dairies according to different technologies (recipes) r_p using different raw materials (milks). The products should be produced in certain quantities to satisfy given product demands. The main issues to be addressed here are to determine several possible implementations of the product demands, the optimal SC routes and quantiles of raw materials and product flows between suppliers, dairies and markets, satisfying the trade-off between environmental and economic objectives, both defined in terms of costs. The latter includes the total SC costs and environmental costs associated with two types of pollution - wastewater assessed as BOD₅ (biochemical oxygen demand for 5 d) generated at each processing task in the production recipe, CO₂ emissions associated with the energy consumed by the dairies and CO₂ emissions produced during transportation. Environmental pollution taxes have been imposed on the dairies to maintain the quantity of the emitted wastewater and CO₂ below acceptable levels.

3. Supply chain model formulation

3.1 Data

The model includes three groups of data: 1). data related to the composition of used raw materials and products; 2). data for the production system, capacities of the milk suppliers, selling prices of milk and products, production costs, distances between milk suppliers, dairies and markets, transportation costs, and payload capacities of the used vehicles; 3). environmental impact data related to the pollutants generated in air and water.

3.2 Deterministic mathematical model

3.2.1. Models of production recipes

The productions of two types of products (P1 and P2) - cottage cheese with low-fat content and high-fat content in two different recipes (PR1 and PR2), each of which uses different raw material - standardized whole milk (RM1) and skimmed condensed milk (RM2) are considered. The production recipes comprise different production tasks performed in units of different types. The first recipe includes three production tasks: milk pasteurization (T1); acidification to produce a raw dairy product (T2); draining to produce the target dairy product

(T3). The second one includes four production tasks: milk dilution (T1); milk pasteurization (T2); acidification (T3) and draining (T4). The mathematical description of the production recipes includes:

The mathematical description of the production recipes includes equations for:

1). The protein, casein and lactose concentrations in the raw materials:

Production recipe 1 (Skimming of whole standardized milk):

$$MP(x(r_p)) = MP \left(1 + \frac{MF - x(r_p)}{CF - MF} \right), \quad r_p = 1, \quad \forall p, p \in P. \quad (1)$$

$$MC(x(r_p)) = MC \left(1 + \frac{MF - x(r_p)}{CF - MF} \right), \quad r_p = 1, \quad \forall p, p \in P. \quad (2)$$

$$ML(x(r_p)) = ML \left(1 + \frac{MF - x(r_p)}{CF - MF} \right), \quad r_p = 1, \quad \forall p, p \in P. \quad (3)$$

Production recipe 2 (Dilution of skimmed condensed milk):

$$MP(x(r_p)) = MP \frac{x(r_p)}{MF}, \quad MC(x(r_p)) = MC \frac{x(r_p)}{MF}, \quad ML(x(r_p)) = ML \frac{x(r_p)}{MF}, \quad r_p = 2, \quad \forall p, p \in P. \quad (4)$$

where MF (%), MP (%), MC (%), ML (%) are the concentrations of milk fat content, proteins, casein and lactose in the milk. CF (%) is cream fat content. $MP(x(r_p))$ (%), $MC(x(r_p))$ (%), $ML(x(r_p))$ (%) are the concentrations of proteins, casein and lactose in the milk. r_p is the recipe used for the production of the dairy product p . $x(r_p)$ is milk fat content, which depend on the production recipe.

2). The products yield $YP(x(r_p))$ (kg) as functions of the fat content in the used raw materials:

$$YP(x(r_p)) = \frac{(RF(x(r_p))x(r_p) + RC_p MC(x(r_p)))RS_p}{PS_p}, \quad r_p = 1,2, \quad \forall r_p, r_p \in R_p, \quad \forall p, p \in P \quad (5)$$

where PS_p (%) is solids content of the products and RC_p (%) and RS_p (%) are the recovery factors for casein, and all solids. $RF(x(r_p))$ (%) is the milk fat recovery factor.

3). Fat in Dry Matter FDM_p (%) used as an indicator for quality of the dairy products.

$$FDM_p = \frac{PF_p}{PS_p}, \quad \forall p, p \in P. \quad (6)$$

where PF_p (%) is the fat content of the product:

Data about processing times and equipment used for implementation of the production tasks and the fractions of the processed raw materials and products are provided in detail in (Kirilova and Vaklieva-Bancheva, 2017).

3.2.2. Supply chain model

Mathematical description of the considered SC includes mass balance equations for the subsystem's suppliers-dairies and dairies-markets to prevent from the accumulation of milk $QM(r_p)$ (kg) in the suppliers and products $QP(r_p)$ (kg) in the dairies.

$$QM(r_p)_i = \sum_{s=1}^S Y(r_p)_{i,s} \cdot \alpha_{i,s}, \quad \forall i, i \in I, \quad \forall r_p, r_p \in R_p, \quad \forall p, p \in P \quad (7)$$

$$QP(r_p)_i = \sum_{m=1}^M X(r_p)_{i,m} \cdot \beta_{i,m}, \quad \forall i, i \in I, \quad \forall r_p, r_p \in R_p, \quad \forall p, p \in P \quad (8)$$

where $Y(r_p)_{i,s}$ (kg) are the quantities of raw materials bought by dairies i from the suppliers s and $X(r_p)_{i,m}$ (kg) are the quantities of products p produced in dairies and sold on markets m . $\alpha_{i,s}$ and $\beta_{i,m}$ are binary variables to structure the SC between suppliers and dairies and dairies and markets.

3.2.3. Model of the supply chain environmental impact

The environmental impact model includes equations for: 1). BOD_5 loads associated with wastewater generated during conducting all production tasks in both recipes and introduced from outside related to the pre-processing of used raw materials. The latter are due to losses of raw materials, by-products and products. They should not exceed predefined eligible levels (Kirilova et al., 2022).

$$BOD_M(x(r_p)) = (0.89x(r_p) + 1.031MP(x(r_p)) + 0.69ML(x(r_p)))10^{-2}, \quad \forall r_p, r_p \in R_p, \quad \forall p, p \in P. \quad (9)$$

$$BOD_P(x(r_p)) = \frac{BOD_M(r_p)}{YP(x(r_p))}, \quad \forall r_p, r_p \in R_p, \quad \forall p, p \in P. \quad (10)$$

$$BOD_{Pa} = 1.5 \cdot 10^{-3}, \quad BOD_{Wh} = 32 \cdot 10^{-3}, \quad \forall r_p, r_p \in R_p, \quad \forall p, p \in P. \quad (11)$$

where $BOD_M(x(r_p)) \left(\frac{kg O_2}{kg milk}\right)$ is the BOD_5 load related to spills of skim milk during implementation of T1 in PR1 and T2 in PR2. $BOD_P(x(r_p)) \left(\frac{kg O_2}{kg product}\right)$ is the BOD_5 load related with losses of products during the implementation of T3 in PR1 and T4 in PR2. $BOD_{Pa} \left(\frac{kg O_2}{kg milk}\right)$ is the BOD_5 load related to deposits of milk on pasteurizers walls during the implementation of T1 in PR1 and T2 in PR2. $BOD_{Wh} \left(\frac{kg O_2}{kg whey}\right)$ is the BOD_5 load related to spills of whey during implementation of T2, T3 in PR1 and T3, T4 in PR2.

The total environmental impact $PBOD(x(r_p))$ (kg) associated with the production of dairy products is:

$$PBOD(x(r_p)) = \sum_{w=1}^W BOD_w \sum_{l=1}^{L(r_p)} m(x(r_p))_{w,l}, \quad \forall r_p, r_p \in R_p, \quad \forall p, p \in P. \quad (12)$$

where $m(x(r_p))_{w,l}$ ($\forall w, w \in W; \forall l, l \in L(r_p); \forall r_p, r_p \in R_p; \forall p, p \in P$) environmental indicators used for determining the quantity of mass of each type of waste w generated in each production task l related to 1 kg product. For their determination In/Out fractions, products yield and the eligible levels of losses have been used (Kirilova et al., 2022).

2). The impact of CO_2 emissions associated with energy consumed for heating and cooling of milk:

$$EIMCO_2(x(r_p)) = \frac{(EH + EC)ECO_2}{CF - MF}, \quad \forall r_p, r_p \in R_p, \quad \forall p, p \in P. \quad (13)$$

where EH and EC is the energy required for the processes of heating and cooling in dairy production (kWh/kg milk). ECO_2 is the mass of CO_2 associated with the energy consumed (kg CO_2 /kWh).

3). Determination of the impact of CO_2 emissions associated with transportation or raw materials $TMCO_2 \left(\frac{kg CO_2}{km \cdot kg milk}\right)$ and products $TPCO_2 \left(\frac{kg CO_2}{km \cdot kg product}\right)$:

$$TMCO_2 = 2 \frac{TCO_2}{VCm} \quad (14)$$

$$TPCO_2 = 2 \frac{TCO_2}{VCp} \quad (15)$$

where $TCO_2 \left(\frac{kg CO_2}{km}\right)$ is the quantity of CO_2 produced by fuel combustion and VCm (kg) and VCp (kg) are the payload capacities of used vehicles for transportation of and products.

3.3 Model constraints

The model includes constraints for: 1). Implementation of the production portfolio in the time horizon; 2). Suppliers capacities; 3). Environmental impact costs that should be paid for the treatment of the generated CO_2 and wastewater pollutants (Kirilova and Vaklieva-Bancheva, 2017).

3.4 Optimization criterion

As an optimization criterion, the SC profit F_{Profit} is used. It is subjected to maximization:

$$F_{Profit} = F_{Revenue} - (F_{PCosts} + F_{MCosts} + F_{TCosts} + F_{BODCosts} + F_{CO_2ECosts} + F_{CO_2TCosts}), \quad \max(F_{Profit}). \quad (16)$$

where $F_{Revenue}$, (BGN) is revenue from the sale of the products in the markets; F_{PCosts} , (BGN) are production costs; F_{MCosts} , (BGN) are costs for purchasing the required quantities of raw materials; F_{TCosts} , (BGN) are costs for transportation of the milk and products; $F_{BODCosts}$, (BGN) are environmental costs paid for treatment of the wastewater generated during the production of the products; $F_{CO_2ECosts}$, (BGN) are environmental costs paid for treatment of CO_2 generated due to the energy consumed from dairy production and $F_{CO_2TCosts}$, (BGN) are environmental costs paid for treatment of CO_2 associated with transportation. The optimization problem is described in details in Kirilova and Vaklieva-Bancheva (2017).

3.5 Robust optimization model

To handle the uncertainties, the approach of Ben-Tal et al. (2005) has been used. The latter is based an extension of already developed deterministic SC model with a Robust Counterpart (RC) for the uncertain products demands. The general formulation of a compact robust optimization problem is following:

$$\begin{aligned} \min \quad & ax + by \\ \text{s.t.} \quad & bx \leq c \\ & bx = dy \\ & a, c, d \in U \end{aligned} \quad (17)$$

where a , c and d are the model parameters that vary in a given uncertainty set U . A vector x is a robust feasible solution to problem if it satisfies all realizations of the constraints from the uncertainty set U . Each uncertain parameter is assumed to vary in a specified closed bounded box as follows:

$$u_{Box} = \{\theta \in R^n: |\theta_t - \bar{\theta}_t| \leq \rho G_t, \quad t = 1, \dots, n\} \quad (18)$$

where $\bar{\theta}_t$ is the nominal value of the θ_t as t_{th} parameter of vector θ . G_t and ρ are positive numbers representing so called "uncertainty scale" and "uncertainty level". According to that RC model can be stated as follows:

$$\begin{aligned} \min \quad & z \\ \text{s.t.} \quad & ax + by \leq z \quad \forall a \in u_{Box}^a \\ & bx \leq c \quad \forall c \in u_{Box}^c \\ & bx = dy \quad \forall d \in u_{Box}^d \\ & y \in \{0,1\}, \quad x \in R^+ \end{aligned} \quad (19)$$

The RC model (19) can be converted to a tractable equivalent model where U_{Box} is replaced by a finite set U_{ext} consisting of the extreme points of U_{Box} , as follows:

$$ax \leq z - by, \quad \forall a \in u_{Box}^a / u_{Box}^a = \{a \in R^{n_a}: |a_t - \bar{a}_t| \leq \rho_a G_t^a, \quad t = 1, \dots, n_a\} \quad (20)$$

The left hand side of inequality (20) contains the vector of uncertain parameters, while all parameters of the right hand side are certain. The tractable form of the above semi-infinite inequality could be written as follows:

$$\begin{aligned} \sum_t (\bar{a}_t x_t + \gamma_t) &\leq z - by, \\ \rho_a G_t^a x_t &\leq \gamma_t, \quad \forall t \in \{1, \dots, n_a\}, \\ \rho_a G_t^a x_t &\leq -\gamma_t, \quad \forall t \in \{1, \dots, n_a\}. \end{aligned} \quad (21)$$

Similarly, the equality and inequality constraints in Eq. (19) can be converted to its tractable equivalent equations through extending the use of the extreme points of the U_{Box} .

4. Results and discussion

The proposed robust optimization approach has been implemented in a real case study. The SC includes two suppliers, two dairies and two markets (M1 and M2). About 30,000 kg per product should be produced over a time horizon of one month. All needed data are given in Kirilova et al. (2022). In the present study, product demands are considered an uncertain parameter. Several deterministic and robust optimization problems have been formulated and solved under nominal data for the product demands and three different uncertainty levels (i.e., $\rho = 0.2; 0.5; 1$).

Table 1: Obtained results from the optimization problems solution for the SC profit

Uncertain SC profit values level	Mean of the SC profit values					Standard deviation of SC profit values	
$\rho = 0$	232,216	234,483	233,676	232,773	232,737	233,177	804
$\rho = 0.2$	232,116	234,449	233,791	232,606	232,737	233,140	852
$\rho = 0.5$	231,968	233,984	233,964	232,356	232,737	233,002	830
$\rho = 1$	231,724	234,316	233,707	231,923	232,738	232,882	1,002

Table 2: Obtained results from the optimization problems solution for the SC total costs

Uncertain level	SC total costs values					Mean of the SC total costs values	
$\rho = 0$	147,351	152,743	152,673	153,739	153,838	152,069	
$\rho = 0.2$	147,031	152,740	152,681	154,201	153,838	152,098	
$\rho = 0.5$	146,605	162,061	152,693	154,893	153,838	154,018	
$\rho = 1$	146,012	152,726	161,597	146,487	153,837	152,132	

Under each uncertainty level, five random realizations have been uniformly generated in the following uncertainty set: $[nominal\ value - \rho G^*, nominal\ value + \rho G^*]$. The deterministic and robust models have been solved using GAMS® optimization software-BARON solver as all calculations have been carried out on an AMD 7 3700X 8-CORE (3.6/4.4. GHz, 32 MB, AM4) CPU with 16 GB DDR4 3600 MHz RAM. Two performance measures have been used to evaluate the models: mean and standard deviation of the obtained results. The optimization problems have been formulated and solved at given boundaries of varying of the products demands, as follows: P1, M1 - 13,000 ÷ 19,000 kg; P1, M2 - 11,000 ÷ 17,000 kg; P2, M1 - 10,000 ÷ 16,000 kg; P2, M2 - 14,000 ÷ 20,000 kg. The results from the optimisation models are listed in Table 1 and Table 2. One can see from Table 1 that the increase in uncertainty level leads to decreasing the SC profit with a relatively

small standard deviation. The lowest mean value of the SC profit of 232,882 BGN has been obtained at the greatest uncertainty level of 1. The results for the SC total costs listed in Table 2 show that they also do not change significantly with an increase in uncertainty level. The largest value of 154,018 BGN has been obtained at an uncertainty level of 0.5. The latter is due to the presence of some higher values for the randomly generated product demands, which are associated with higher costs and lower profit. In general, the obtained results show a relatively uniform level of profit values with varying product demands. For this reason, the model can be considered sustainable.

5. Conclusions

This study proposes an approach for the robust optimal design of dairy SC operating under uncertain product demands. Deterministic and robust optimization problems under different random realizations were formulated and solved under nominal data for the product demands at three different uncertainty levels. The nominal data for the product demands were randomly generated using uniform random distributions in a predefined uncertainty set. Two performance measures were used to evaluate both the robust and deterministic models: the mean and standard deviation of the objective function values under random realizations. The approach was implemented in a real case study from Bulgaria. It comprises the production of two types of dairy products that are produced in two dairies according to two recipes in which two different types of milk are used. The latter are provided by two suppliers. The produced products are sold in two markets. The obtained results show that the increase in the uncertainty level leads to a decrease in the SC profit with a relatively small standard deviation. At the greatest uncertainty level of 1, the lowest mean value of the SC profit of 232,882 BGN was obtained. The results for the SC total costs show that they also do not change significantly with an increase in the uncertainty level with the exception of the uncertainty level of 0.5 in which the total costs have the largest value of 154,018 BGN. This is due to some higher values for the randomly generated product demands, which are associated with higher costs and lower profit.

Acknowledgments

This study was carried out with the financial support of National Science Fund, Ministry of Education and Science of the Republic of Bulgaria, Contract No. КП-06-Н37/5/06.12.19.

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