

A T-Spherical Fuzzy Set Approach to Develop a Fuzzy Ranking Index for Water Resource Management in the Philippines

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A water poverty index (WPI) is a multi-criterion indicator that assesses water stress and scarcity that integrates the physical estimates of water availability with the socioeconomic drivers of poverty. However, the decision-making process can be a participatory, iterative process that is characterized by uncertainty due to ambiguity, impreciseness, or incomplete information. This study focuses on the development of a fuzzy ranking index based on the T-Spherical Fuzzy Set under a multi-criteria framework. An illustrative case study on measuring the WPI is presented in the ranking of the vulnerability of a municipality, in Luzon Island, Philippines. In this assessment, the main contributors to water poverty in the area include water availability, lack of access to safe water and capacity to manage the development of water sources. This linguistic approach to score indicators was found appealing and took into consideration the uncertainty in assigning linguistic assessments by participants. There remain other aspects of the decision-making process that can be improved such as compositing several sub-indicators of each WPI; building consensus among multi-stakeholders and using the same process to rank alternatives to reduce water poverty.

1. Introduction

In the next decades, freshwater scarcity can increase due to accelerated population growth, climate change, intensive farming, technological advancement, and the inevitable water pollution (Jha et al., 2014). Majority of the countries worldwide such as China, India, Mexico, Pakistan, South Africa, and the Philippines deal with the scarcity of water and its underlying consequences (FAO, 2012). This results in a reduction of the capacity for the water supply in the agricultural sector while simultaneously satisfying the needs in the domestic and industrial water demands. Moreover, additional research and resources are essential to focus on the efficient utilization of probable water sources and a strategic allocation of water (Terêncio et al., 2018).

A holistic tool that is designated to contribute to effective water management is measured through the water poverty index (WPI) (Sullivan et al., 2003). This has been regarded to be a useful contribution to a suite of tools available to enhance the usefulness of managing water at a community level. It considers different data sources including water availability (e.g., access, local water resources use), economic and social capacity, and water quality to be utilized by water development agencies and the local people that can monitor the progress in the provision of water at the community level. A set of community-based indicators can be a basis to provide specific international agencies and countries that can guide towards the creation of various policies. These indicators are then used to compute the WPI through weighted arithmetic mean. The WPI introduced in Sullivan et al. (2003) is widely accepted and has also applied in various regions such as that of Nepal (Koirala et al., 2020) and Taiwan (Chen et al., 2020). However, there are also some challenges in using WPI such as the arbitrariness of weighting, lack of available data to measure reliably WPIs particularly in low-to-middle income countries and possible loss of information during the aggregation process.

Thus, this study proposes a novel method of measuring WPI to rank water scarce and stress areas that simultaneously incorporates the socioeconomic drivers of poverty and the physical estimates of water availability in the Philippines. The fuzzy ranking index method is built from multi-criteria framework that uses T-Spherical Fuzzy Set (T-SFS) to model the uncertainty attributed to linguistic assessment despite limited availability of quantitative data. For example, a recent extension of fuzzy set in a form of T-SFS was used for the multi-attribute selection of solar cells for renewable energy resources (Zeng et al., 2019). Given our proposed method, the linguistic assessment to measure WPI under uncertainty aims to provide a composite index that can be readily calculated and comprehensible to inform water resource managers at the local scale even in the low-income setting. In addition, weighted geometric mean is used as an alternative aggregation operator since weighted arithmetic mean of the original WPI suggests complete compensability among the components of the indicator. WPI based on weighted arithmetic allows offsetting the poor performance in some indicators by sufficiently high values of other indicators. On the other hand, WPI based on weighted geometric mean is partially compensatory approach wherein poor performance in some indicators could be penalized more heavily.

The arrangement of the specific section in this paper is as follows. Section 2 indicates the associated methodology of the T-SFS and the description of the case study. This is followed by the discussion of the results in WPI of the case study in the Philippines in Section 3 while Section 4 gives the concluding statements and the recommendations for the future research direction in this study.

2. Methodology

The concept of T-Spherical fuzzy set was first introduced in Mahmood et al. (2019) to model the ambiguous human opinion as a generalization of Zadeh’s fuzzy set and its extension such as that of intuitionistic fuzzy set and picture fuzzy set. This section thus introduces the definitions related to spherical fuzzy set and its generalization, T-spherical fuzzy set, and then describe how WPI is computed from the T-Spherical fuzzy sets.

Definition 1. Let X be in a finite domain and $x \in X$. T-spherical fuzzy set (T-SFS) is defined as: $T = \{x, \mu(x), \nu(x), \pi(x) \forall x \in X\}$ with the condition that $0 \leq Sum(\mu^t, \nu^t, \pi^t) \leq r^t \forall t \in Z \geq 1$. Here three components $\mu, \nu, \pi: X \rightarrow [0,1]$ represents the degree of membership, degree of non-membership, and degree of indeterminacy, respectively. Z refers to positive integers and $r^t \rightarrow [1, 3^{1/t}]$ wherein a particular case of T in X , for example is a spherical fuzzy set (SFS) at $t = 2$ with the condition of $0 \leq Sum(\mu^2, \nu^2, \pi^2) \leq 1$, i.e., $0 \leq \mu^2 + \nu^2 + \pi^2 \leq 1$. For ease of computation, T-spherical fuzzy number is designated as an ordered triple: $\tilde{T}_s = (\mu_{\tilde{T}_s}, \nu_{\tilde{T}_s}, \pi_{\tilde{T}_s})$.

Definition 2. TSWGM is an aggregation operator for n T-spherical fuzzy numbers using weighted geometric mean such that the weight vector $w_i \in [0,1]$; $\sum_{i=1}^n w_i = 1$

$$TSWGM(\tilde{T}_{s1} \dots \tilde{T}_{sn}) = \Pi_{i=1}^n (\tilde{T}_{si})^{w_i} = \left\{ \Pi_{i=1}^n \mu_{\tilde{T}_{si}}^{w_i}, \left[1 - \Pi_{i=1}^n (1 - \nu_{\tilde{T}_{si}}^{w_i}) \right]^{\frac{1}{t}}, \Pi_{i=1}^n \pi_{\tilde{T}_{si}}^{w_i} \right\} \quad (1)$$

Definition 3. Defuzzification of T-spherical fuzzy number is defined as follows:

$$Score(\tilde{T}) = 1 - \left[\frac{1}{3} \{ (1 - \mu^t)^\beta + (\nu^t)^\beta + (\pi^t)^\beta \} \right]^{\frac{1}{\beta}} \quad (2)$$

where $\beta \geq 1$ is the distance parameter. Here the $Score(\tilde{T}) \rightarrow [0,1]$.

Proposed Method for Computing WPI using T-Spherical Fuzzy Set are as follows:

Step 1: Evaluate the areas with respect to the n indicators of WPI using the linguistic scale listed in Table 1.

Table 1: Linguistic scale for the indicators of WPI

Linguistic scale	Symbol	μ	ν	π	Score
Exceptionally Low	EL	0.900	0.100	0.100	0.880
Very Low	VL	0.800	0.200	0.250	0.771
Moderately Low	ML	0.700	0.300	0.350	0.672
Low	L	0.600	0.400	0.400	0.585
Satisfactory	S	0.500	0.500	0.500	0.500
High	H	0.400	0.600	0.400	0.438
Moderately High	MH	0.300	0.700	0.350	0.373
Very High	VH	0.200	0.800	0.250	0.236
Exceptionally High	EH	0.100	0.900	0.100	0.157

Step 2: Compute WPI of each area using Eq(3):

$$\bar{WPI} = \Pi_{i=1}^n (\bar{WPI}_i)^{w_i} \quad (3)$$

where \overline{WPI}_i is the rating of the area for indicator i such that the indicator weight $w_i \in [0,1]$; $\sum_{i=1}^n w_i = 1$. Since the rating is a T-spherical fuzzy set, use Eq(1) as the aggregation operator.

Step 3: Compute WPI score of each area from defuzzification of \overline{WPI} using Eq(2).

Step 4: Rank the area using the score obtained from Eq(2) using $t = 2$ and $\beta = 19/8$. The closer the value of the score to 1 indicates high risk of water poverty index. Likewise, the closer the value of the score to 0 indicates low risk of water poverty index.

3. Results and discussion

Our study area is Mulanay which is a municipality located in the Province of Quezon, island of Luzon, Philippines with geographic coordinates of 13°31'20"N 122°24'15"E. It is subdivided into 28 administrative areas called *barangays*. The municipality is an agricultural town and has a wide coast facing Tayabas Bay. The total land area is about 420 km². As of 2020, the total population was 55,576 people (PSA,2020) giving it a population density of about 132 persons/km². Except for the town center (Poblacion), communities or population areas are dispersed and developed a linear pattern following roads.

In the Philippines, water service levels are classified under three types, depending on the method by which the water is made available to the consumers (The World Bank, 2012). Level I refers to a point source system. This level provides a protected well or a developed spring with an outlet, but without a distribution system. A Level I facility serves an average of 15 households within a radius of 250 meters. In this type of system, the users go to the source to fetch the water. Level II refers to a communal faucet system or stand posts. This type of system is composed of a source, a reservoir, a piped distribution network, and communal faucets. Usually, one faucet serves four to six households within a radius of 25 meters. Level III refers to a waterworks system or individual house connections and this includes a source, a reservoir, a piped distribution network, and individual household taps. It is suited for densely populated urban areas where the population can afford individual connections.

In the Municipality of Mulanay, the barangays facing the southwestern side lie along the coastline of Tayabas Bay and includes the Población. These are served by Level II and III systems. The remaining barangays, generally have a more dispersed population, are served by Level I and II systems.

Water source is generally groundwater and extracted through springs and wells (i.e., shallow, and deep). Rainwater collection is also practised by households and has not been scaled up to the community level. Community access to water, in general, is more challenging for barangays with dispersed populations and when served by poorly maintained shallow wells (a Level 1) and fewer spring sources.

To ensure that major issues related to water are considered, the WPI considered five (5) key components (Sullivan et al, 2003) and described in Table 2. Each of the components can be further evaluated as quantitative or qualitative subcomponents and indices based on the available data.

Data utilized for the linguistic scoring scale were mainly drawn from municipal social, economic, and physical profiles which provided information on threshold in this study. Barangays with higher WPI values are identified in the ranking process and a selection is drawn among them (e.g., top five). water accessibility, water demand, water sources (e.g., rainfall, wells, springs), health concerns, household incomes and economic conditions.

Table 2: Water poverty indicators and their descriptions

Indicators	Symbol	Description
Resource availability of water	R	Physical availability of water, taking into account the variability of its quality and volume
Accessibility of safe water and sanitation	A	The extent of accessibility of water for human use such as the distance to the source, time needed to collect water etc.
Capacity to manage water resource	C	The people's ability to avail water in the sense of income to allow purchase of improved water; or proxy indicators like education or availability of skilled labor, which interact with income and indicate a capacity to lobby for and manage a water supply.
Use of water in terms level of service	U	The different means by which water is utilized, such as agricultural, domestic and industrial.
Environmental Integrity of water sources	E	Quality of surrounding environment that influences water quality at supply source (e.g., sources and pathways of contaminants)

Each of the indicators is standardized using Steps 1 and 2, such that it falls in the range of zero to one. This also results in a WPI value between zero to one, with zero being the lowest possible level of poverty, and one

being the highest value of water poverty. A value of $WPI = 0.5$ is taken as a critical threshold. The linguistic approach to score indicators is generally appealing, and the fuzzy approach takes into consideration the uncertainty made by participants in assigning linguistic values, shown in Table 1. The utility of a T-Spherical fuzzy set approach to develop a fuzzy ranking index that brings these considerations to WPI water development needs to be further studied. An example of the output from this exploratory study is shown in Figure 1.

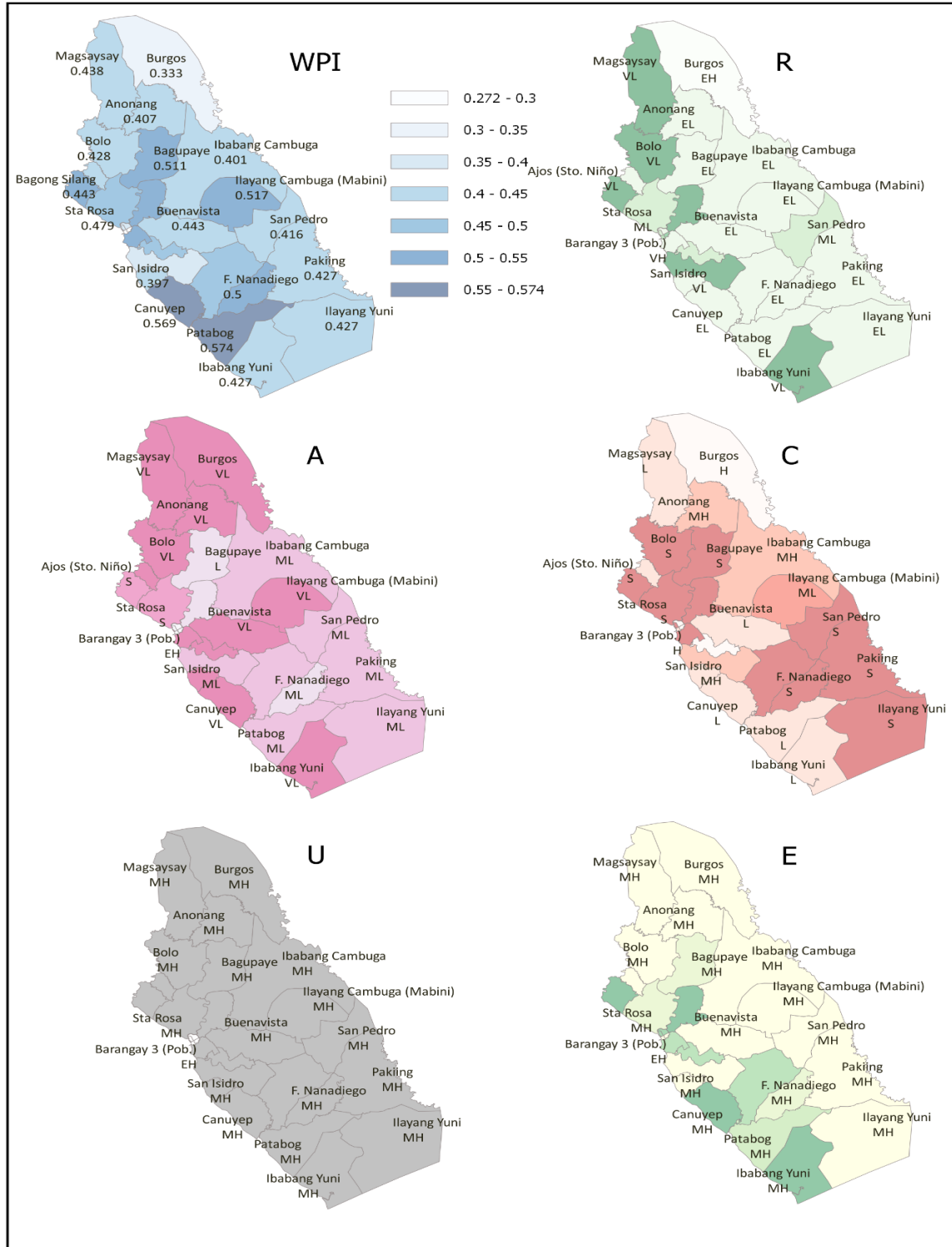


Figure 1: Water poverty index (WPI) and indicator values for resource availability (R), accessibility (A), capacity (C), use of water (U), and environmental integrity (E) for selected barangays

In this study, each indicator weight w_i is given a value of 0.2, assuming an equal importance of the five indicators in compositing WPI. Rating assignments given to barangays across WPI indicators are shown in Table 3. Table 4 is an example of the conversion of the linguistic rating to T-spherical fuzzy set. \overline{WPI} of each barangay is computed from weighted geometric mean of these fuzzy ratings along the row in Table 4. The T-spherical fuzzy set (\overline{WPI}) in the last column in Table 4 is obtained using the aggregation operator described in Eq(1). Note that the T-spherical fuzzy number at $t = 2$ degenerates to spherical fuzzy set which follows the following condition: $0 \leq \mu^2 + \nu^2 + \pi^2 \leq 1$.

Illustrative results of the calculated WPI and linguistic rating for the five (5) indicators are shown in Figure 1 while Table 5 shows the list of Barangays with WPI greater than or equal to 0.5. Note that the WPI score in Table 5 is obtained from defuzzification of the T-spherical number using Eq(2).

Table 3: Sample rating of linguistic assess for the six sample barangays

Location	Indicators				
	R	A	C	U	E
Ajos	VL	S	S	MH	S
Amuguis	ML	VL	H	MH	MH
Anonang	EL	VL	MH	MH	EH
Bagong Silang	EL	VL	L	MH	EH
Bagupaye	EL	L	S	MH	H
Barangay 1	ML	EH	S	EH	ML

Table 4: Conversion to T-spherical Fuzzy Set to compute \overline{WPI}

Location	Indicators					\overline{WPI}
	R	A	C	U	E	(μ, ν, π)
Ajos	(.80,.20,.25)	(.50,.50,.50)	(.50,.50,.50)	(.30,.70,.35)	(.50,.50,.50)	(.496,.520,.405)
Amuguis	(.70,.30,.35)	(.80,.20,.25)	(.40,.60,.40)	(.30,.70,.35)	(.30,.70,.35)	(.458,.566,.336)
Anonang	(.90,.10,.10)	(.80,.20,.25)	(.30,.70,.35)	(.30,.70,.35)	(.10,.90,.10)	(.365,.676,.198)
Bagong Silang	(.90,.10,.10)	(.80,.20,.25)	(.40,.60,.40)	(.30,.70,.35)	(.10,.90,.10)	(.419,.633,.204)
Bagupaye	(.90,.10,.10)	(.60,.40,.40)	(.50,.50,.50)	(.30,.70,.35)	(.40,.60,.40)	(.504,.522,.309)
Barangay 1	(.70,.30,.35)	(.10,.90,.10)	(.50,.50,.50)	(.10,.90,.10)	(.70,.30,.35)	(.300,.729,.228)

Table 5: Sample results for the fuzzy ranking of barangay with WPI greater than or equal to 0.5

Location	μ	ν	π	WPI Score
Patabog	0.584	0.449	0.287	0.574
Canuyep	0.579	0.460	0.281	0.569
Ibabang Yuni	0.565	0.465	0.337	0.557
Latangan	0.514	0.504	0.388	0.517
Bagupaye	0.504	0.522	0.309	0.511
Matataja	0.504	0.522	0.309	0.511
Butanyog	0.504	0.535	0.274	0.510
Ajos	0.496	0.520	0.405	0.503
F. Nanadiego	0.490	0.542	0.293	0.500

Note that Figure 1 presents the WPI values for the 28 barangays. Nine (9) barangays with a total population of 19,614 (or 35 %) have a WPI greater than 0.5 (beyond the threshold and indicate vulnerability) and these are the following (see Table 5): Patabog, Cayunep, Ibabang Yuni, Latangan, Bagupaye, Matataja, Butanyog, Ajos, and F. Nanadiego.

In terms of resource availability (R), more than half of the total number of barangays (i.e., 15) have exceptionally low (EL) water availability. Barangays in mountainous or remote areas are typically served by Level 1 and 2 water systems where sources include shallow wells and springs. Moreover, most of these barangays have moderately low to low accessibility (A). The members of the households would normally walk a few meters to a

kilometer or even a few kilometers just to get their water needs. Only one barangay, i.e., Burgos is classified to have an exceptionally high-water resource availability.

In terms of capacity to manage water resources (C), twenty barangays have scored between satisfactory (S) to moderately high (MH). On the other hand, MH to H utilization of water (U) in different aspects namely domestic and agricultural sectors are observed. The main source of income in the whole municipality is agriculture thus a high level is observed. The Patabog, Canuyep and Ilayang Cambuga have exceptionally low to low ratings on resource availability, accessibility and capacity. Latangan and Bagupaye have satisfactory to high ratings for capacity compared to the three barangays.

Future work will then consider the effect of weighting of the indicators on the robustness of ranking results. For example, spherical fuzzy analytical hierarchy process (Kuok and Promentilla, 2021) can be used to derive weights from the elicited value judgment of these decision makers.

4. Conclusions

The study utilized a methodology of the T-Spherical Fuzzy Set to develop the Water Poverty Index representing five key components to index and rank the water poverty for twenty-eight barangays of the municipality of Mulanay, Quezon, Philippines. In the preliminary assessment, the main contributors to water poverty in the area are water availability, lack of access to safe water and capacity to manage development of water sources. This linguistic approach to score indicators is appealing and takes into consideration the uncertainty in assigning linguistic assessments by participants; however, there remains other aspects of the decision-making process that can be improved. For example, in compositing several sub-indicators of each main indicator R, A, C, U and E; in building consensus among multi-stakeholders, in validating results and in utilizing a similar approach to rank alternatives to reduce water poverty. These are the areas that will be further studied to make the method for decision-making more robust and practical for users.

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