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A New Tray Efficiency Model: How Simple May It Be?

Markus Duss[a,*], Ross Taylor[b]

[a]Sulzer Chemtech Ltd, Winterthur, Switzerland
[b]Clarkson University, Potsdam, New York, USA
markus.duss@sulzer.com

Engineers rarely apply the most rigorous tray efficiency models due to the significant effort required to implement them and due to their relative lack of accuracy. Rigorous traditional models require a good deal of information; and mechanistic models increase the complexity even further. In sharp contrast, O'Connell's empirical but simple correlation is widely applied despite its shortcomings.
This paper presents a simplified model based on the traditional approach that allows a process engineer to estimate the tray efficiency for sieve trays during the hydraulic design phase, when an equilibrium stage simulation is available. Despite the simplifications, the new model is more accurate than more modern methods and the O'Connell method.

1. Introduction

The more traditional tray efficiency models, an example being that of Chan and Fair (1984), are based on a model or correlation for the point efficiency that is assumed constant over the tray. This, and other, rigorous models include correlations to account for the tray hydraulics to predict froth height, clear liquid height, liquid entrainment, weeping, etc. Despite this effort, the accuracy of these models is still not what would be desired. Moreover, generalized correlations cannot account for proprietary tray design features that aim to improve capacity and/or efficiency and this restricts the applicability of rigorous efficiency models.
Detailed mechanistic models, such as those by Prado and Fair (1990) and by Garcia and Fair (2000), take this idea even further by dividing the froth height into 'hole activity', 'bulk froth', and 'spray' zones and in addition distinguishing 'jetting', 'large bubbling', and 'small bubbling' zones. Each of these zones has its own "local" point efficiency and all are required to get the tray efficiency.
Despite the considerable effort that has gone into the development of these and other similar efficiency models they still are lacking in the accuracy that would allow process engineers to rely on their outcome.
In complete contrast, O'Connell's older empirical correlation (O'Connell, 1946) might be the most often-applied model, due, in part, to its simplicity and because its prediction is often conservative, so that column designs based on O'Connell are more likely to achieve the required performance.
This paper presents a new model for predicting the efficiency of cross-flow sieve trays based on a simplified version of the traditional modelling approach, achieving an improved accuracy despite the significantly reduced effort. The model uses only liquid and vapor viscosities and simple tray geometrical features to represent mass transfer coefficients and interfacial area. Further, the model assumes the point efficiency to be independent of throughput when the vapor to liquid ratio remains constant. This is, in many cases, an acceptable assumption for columns operated within their design limits.

2. New Model – a Simplified Traditional Approach

The section efficiency is the number of theoretical stages found in a section divided by the number of actual trays:

$$\eta_{Section} \equiv \frac{n_{th}}{n_{trays}} \tag{1}$$

The number of theoretical stages derived from equilibrium stage simulations is the basis for determining the number of actual trays and therefore, any attempt quantifying mass transfer for cross-flow trays must yield the section efficiency. The section efficiency, also referred to as the overall efficiency, can change from stage to stage and is not necessarily constant within a column section, as the name might imply.
Lewis (1936) has presented a fundamental approach to obtain the section efficiency by using the Murphree point efficiency (usually based on the number of transfer units) to predict the performance of cross flow trays, yielding the Murphree tray efficiency by integrating over the flow path length. The section efficiency is then determined as a function of the Murphree tray efficiency and the stripping factor λ, which is the ratio of the slopes of operating and equilibrium lines, as given by Eq(2).

$$\lambda = \frac{m}{L/G} \tag{2}$$

The new model, based on a simplification of the traditional approach, is using the following four equations:

$$\frac{1}{N_{OG}} = \frac{1}{N_G} + \frac{\lambda}{N_L} \tag{3}$$

Equation (3) is the sum of the vapor and liquid side mass transfer resistances for a given froth height and yields the overall number of transfer units for the gas/vapor phase. Throughput of vapor and liquid, mass transfer coefficients, interfacial area and froth height are required to determine N_G and N_L in a rigorous way. All rigorous models spend most of their effort determining the number of transfer units. The model presented here is based on an assumption that the N_G and N_L are independent of throughput; this leads to a considerable reduction of effort.

$$\eta_{Point} = 1 - e^{-N_{OG}} \tag{4}$$

Equation (4) gives the Murphree point efficiency for the vapor phase and is derived assuming that the vapor is in plug flow whereas the liquid is vertically well mixed. The point efficiency expresses the fractional approach to equilibrium at a given position (point) on the tray. Given the underlying assumed flow patterns, the point efficiency always is smaller than unity.

$$\eta_{Tray} = \frac{e^{\lambda \cdot \eta_{Point}} - 1}{\lambda} \cdot \left(1 - e^{-c_1 \cdot FPL}\right) \tag{5}$$

The Murphree tray efficiency is retrieved from Eq(5) which is defined with respect to the gas phase in equilibrium with the liquid leaving the tray and the average vapor concentrations entering and leaving the tray. The Murphree tray efficiency typically is smaller than unity but can be, at least theoretically, greater than unity, depending on the point efficiency and the stripping factor. Lewis derived Eq(5) using the following assumptions: a) the point efficiency is constant across the tray; b) the liquid travels in plug flow across the tray; c) the vapor concentration is uniform at the inlet to the tray (Lewis case 1). Assuming the vapor concentration to be uniform at the tray inlet actually holds true only for the bottom tray where the vapor inlet either is from a reboiler or is a feed stream. This assumption considerably simplifies the method compared to Lewis case 2 or 3, which actually would be required for parallel-flow or cross-flow trays, respectively. However, the resulting difference between Lewis case 1 and 3 is negligible, particularly for point efficiencies below 0.8. For the sake of simplicity, Lewis case 1 is adopted here. Lewis did not include any correction for the flow path length, FPL, since integration from down-comer inlet to outlet inherently assumes an infinite number of flashes in the flow path. We included the flow path length in Eq(5) and adapted an empirical correction factor c_1, similar to proposals by other authors (e.g. Gautreaux and Connell, 1955).

$$\eta_{Section} = \frac{\ln\left[1 + \eta_{Tray} \cdot (\lambda - 1)\right]}{\ln(\lambda)} \tag{6}$$

Finally, Eq(6) provides the section efficiency based on the Murphree tray efficiency and the stripping factor by converting driving forces to equilibrium stages. Independent of the rigor of the model, Eq(6) remains the same and only the slopes of equilibrium and operating lines are relevant.
The simplified approach represented by Eq(3) to Eq6) requires regressing N_G, N_L and c_1. The stripping factor is system dependent and defined by vapor-liquid equilibrium of the components involved and the operating conditions; no other parameter is required.
N_G and N_L are defined as follows (Lockett, 1986):

$$N_G \equiv \frac{k_G \cdot a_I \cdot h_f}{G} \quad \text{and} \quad N_L \equiv \frac{k_L \cdot a_I \cdot h_f}{L} \tag{7}$$

Assuming that the numbers of transfer units are independent of the tray hydraulics and remain constant for a given system, allows the interpretation of these quantities in terms of mass transfer coefficients and interfacial area (implicitly including hydraulics), and not explicitly using liquid and vapor throughput or froth height.

3. Data Base

In order to regress N_G, N_L and c_1, we used the database compiled by Chan (1983), which includes some 200 data points for the following systems: cyclohexane – n-heptane (C6/C7), iso-butane – n-butane (iC4/nC4), ethylbenzene – styrene, water – acetic acid, methanol – water and ammonia – water (absorption). Chan's database includes the stripping factor, tray geometries and hydraulic information, namely the operating capacity. Garcia (1999) added the systems water – isopropanol (water/IPA), n-octanol – n-decanol (n-C8/ n-C10) and o-xylene – p-xylene to Chan's database, which we have included in the analysis. Only data points below 98% and above 25% capacity were used since our approach does not consider liquid entrainment or weeping. The database includes only data obtained with sieve trays. Tables 1 and 2 summarize the data.

Table1: Physical properties and stripping factor

Vapor viscosity cP		Liquid viscosity cP		Vapor density kg/m^3		Liquid density kg/m^3		Surface tension mN/m		Stripping factor -	
min.	max.	min.	max.	min.	max.	min.	max.	min.	max.	min.	max.
0.0065	0.0177	0.05	1.5	0.05	89	373	998	1.1	72	0.1	8.6

Table 2: Tray geometry (sieve trays)

Column diameter m		Flow path length m		Frac. open area hole / bubbl. area		Hole diameter mm		Weir height mm		Tray spacing mm	
min.	max.	min.	max.	min.	max.	min.	max.	min.	max.	min.	max.
0.457	1.22	0.24	0.965	0.038	0.14	3.2	12.7	14.3	50.8	305	610

4. Parameter Regression

The mass transfer coefficients could be correlated best using the viscosities with the following dependencies:

$$k_G \propto \mu_G^{-0.3} \quad \text{and} \quad k_L \propto \mu_L^{-0.5} \tag{8}$$

The interfacial area is, as per the regression, best represented by the liquid viscosity and using the tray geometrical features fractional open area, hole diameter and weir height:

$$a_I \propto \mu_L^{-0.1} \cdot FOA^{-0.2} \cdot D_{Hole}^{-0.1} \cdot h_{Weir}^{0.05} \tag{9}$$

Finally, the constant of proportionality for the vapor and liquid side was regressed. The equations below use ratios based on reference values for the properties and geometrical features. The units used are cP for the viscosities, mm for the hole diameter and the weir height, and FOA is defined to be the total hole area divided by the bubbling area.

$$N_G = 1.3 \cdot \left(\frac{\mu_G}{0.01}\right)^{-0.3} \cdot \left(\frac{\mu_L}{0.2}\right)^{-0.1} \cdot \left(\frac{FOA}{0.08}\right)^{-0.2} \cdot \left(\frac{D_{Hole}}{12}\right)^{-0.1} \cdot \left(\frac{h_{Weir}+5}{55}\right)^{0.05} \tag{10}$$

$$N_L = 5 \cdot \left(\frac{\mu_L}{0.2}\right)^{-0.6} \cdot \left(\frac{FOA}{0.08}\right)^{-0.2} \cdot \left(\frac{D_{Hole}}{12}\right)^{-0.1} \cdot \left(\frac{h_{Weir}+5}{55}\right)^{0.05} \tag{11}$$

The weir height correction includes a summand of 5 mm in the numerator. This value was not determined based on regressed data but was included in order to achieve a drop in efficiency of 10% to 15% if no weir was present; this better reflects experience.

The only missing parameter is the constant c_1 in Eq(5) for the flow path length (in meters). Our regression achieved the best match to data with:

$$c_1 = 5 \tag{12}$$

The regressed values are in line with theoretical considerations and experience. If the vapor and liquid viscosity were equal to their reference values, N_L would be almost 4 times higher than N_G, indicating that the system is predominantly vapor side controlled (as long as the stripping factor is not far from unity). Increased liquid viscosity will affect mainly liquid side mass transfer (diffusivity) but also negatively affects the interfacial area. The outcome that the efficiency decreases with increasing fractional open area and hole diameter is commonly accepted by tray experts, also that the efficiency weakly increases with increasing weir height. Finally, the finding that the efficiency improves with increasing flow path length is generally accepted and Eq(5) reflects this for the tray efficiency. The section efficiency did not show any systematic dependency on the weir loading, confirming that the assumption not to include explicitly hydraulic behavior is acceptable.

5. Results

We calculated the section efficiencies using Eq(3) to Eq(6) along with Eq(10) to Eq(12). Figure 1a shows the parity plot for data points having capacities below 98% and above 25%. This was the range found where liquid entrainment or weeping had no major impact on the measured section efficiency. The average absolute deviation is below 10% and the average relative deviation is -3.2%. The latter indicates that the calculated efficiencies are on average only slightly conservative. We determined the deviations relative to measured values, i.e. (calculated – measured)/measured.
Figure 1b includes data points outside the applied hydraulic limits and the model achieves a significantly reduced accuracy for these points. It clearly indicates that any modelling approach not considering tray hydraulics can achieve only reasonable accuracy when applied within the appropriate operating window.

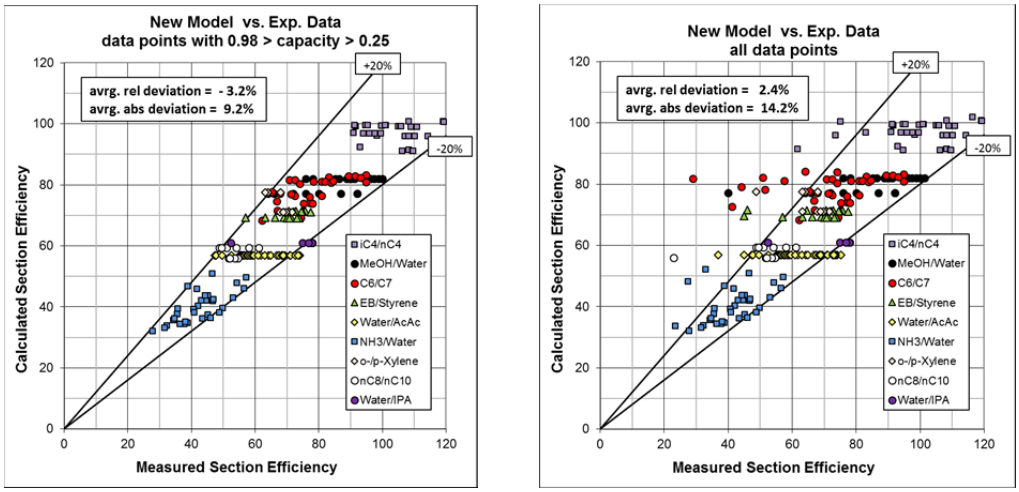

Figure 1a: Parity plot of calculated and measured section efficiency, using only data points below 98% and above 25% capacity. Figure 1b: including the data points outside the applied operating limits.

6. Comparison with Other Models

For comparison purposes, the models by Chan and Fair (1984) and by Garcia and Fair (2000) were used, the latter using the mechanistic modelling approach described by Prado and Fair (1990).
In addition, we also compare the outcome of O'Connell's method, represented with the following simple relation:

$$\eta_{Section} = 0.503 \cdot (\mu_L \cdot \alpha)^{-0.226} \tag{13}$$

Various authors have proposed modifications to O'Connell. For present purposes, we used a recently published modification (Duss and Taylor, 2017) which uses the stripping factor instead of the relative volatility and a modified exponent. (The modifications were based on theoretical considerations.)

$$\eta_{Section} = 0.503 \cdot \mu_L^{-0.226} \cdot \overline{\lambda}^{-0.08} \tag{14}$$

where $\overline{\lambda}$ is defined as follows: if $\lambda > 1$ then $\overline{\lambda} = \lambda$; if $\lambda < 1$ then $\overline{\lambda} = 1/\lambda$.

Table 3 shows the outcome of the section efficiency for O'Connell and its modification for data points within the hydraulic limits. Unlike O'Connell's original method, this modification can be used for absorption and stripping applications. O'Connell's original method achieves an average relative deviation of -17% and, therefore, predicts conservative section efficiencies. Eq(14) predicts more optimistic section efficiencies, -3%, and the average absolute deviation is 14% compared to 20% for O'Connell. The modification predicts the section efficiency for absorption of ammonia with water reasonably well.

Table 3: Section efficiency – deviations from measured values
Values in Table 3 are based on (calculated – measured) / measured

O'Connell		mod. O'Connell		New Model	
avrg. rel. dev.	abs. rel. dev.	avrg. rel. dev.	abs. rel. dev.	avrg. rel. dev.	abs. rel. dev.
-17%	20%	-3%	14%	-3.2%	9.2%

Chan and Fair (1984) and Garcia and Fair (2000) did not report a comparison of their models to experimental section efficiencies, rather they compared their models to point efficiencies. To this end, these authors back calculated the point efficiency for each of the experimental data points; this requires making assumptions about the tray's hydraulic behavior. The step to calculate the point efficiency is not straightforward and they used a method based on the Peclet number, which includes flow path length, liquid phase eddy diffusivity, and mean liquid residence time.
In view of their indirect approach, a proper comparison of their methods with the model of this paper is not possible and we used therefore two different approaches for the benchmarking:
Approach 1: from the experimentally reported section efficiencies, the "measured" point efficiencies are back calculated using Eq(6) and (5). These values are compared to predicted values obtained with Eq(4).
Approach 2: "measured" point efficiencies reported by Garcia (1999) are compared to the outcome of Eq(4).
Table 4 shows the outcome of the point efficiency for the rigorous models and the new model. The average absolute error of the presented model is 6.9% when back calculating point efficiencies from section efficiencies and the conclusion is that point efficiencies are less prone to error.

Table 4: Point efficiency – deviations from "measured" values
Values in Table 4 are based on (calculated – measured) / calculated as reported by Garcia and Fair (2000)

Chan & Fair		Garcia & Fair		New Model[1)]		New Model[2)]	
avrg. rel. dev.	abs. rel. dev.	avrg. rel. dev.	abs. rel. dev.	avrg. rel. dev.	abs. rel. dev.	avrg. rel. dev.	abs. rel. dev.
8.4%	18.2%	-1.3%	13.3%	-3.0%	6.9%	-4.5%	11.0%

[1)] "measured" point efficiency retrieved from measured section efficiency, using Eq(6) and Eq(5)
[2)] "measured" point efficiency retrieved with Garcia's method using the Peclet number

7. Conclusions

In the title, we raised the question: how simple may it be? Based on the analysis of the experimental data points and the benchmarking, we recommend using the stripping factor to determine tray efficiencies. O'Connell's original correlation does not require the stripping factor but uses the relative volatility instead. However, his correlation has significant shortcomings and we recommend using methods that are more sophisticated. The modified O'Connell method uses the stripping factor, yields better agreement with experimental data and, in addition, is applicable to absorption and stripping operations. This method provides a rough estimate for the section efficiency.
Using rigorous traditional or mechanistic models might remain the ultimate goal, provided the models have an improved accuracy that has not yet been achieved. Applying rigorous models requires a rate based approach, determining all relevant properties during simulation time for each tray. In addition, all tray details must be available and this might add an additional loop to the design procedure.
The model presented here achieves acceptable results, better than are obtained using the rigorous models investigated in this paper. Design engineers can readily obtain all properties, except the stripping factor, from an equilibrium stage simulation during the hydraulic design phase, once the tray geometry is determined. The stripping factor requires the slope of the equilibrium line, which is troublesome to retrieve for multicomponent systems. The method described by Taylor and Duss (2017) is recommended for calculating the slope of the equilibrium line.
Finally, to make the model applicable to valve trays, we recommend including a factor in Eq(10) and Eq(11) to account for the altered point efficiency, provided experimental data are available to confirm the outcome.

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Symbols

a_I	(m^2/m^3)	Specific vapor-liquid interfacial area
c_1	(1/m)	Constant of proportionality for FPL in Eq(5)
D_{Hole}	(mm)	Sieve hole diameter
FOA	(-)	Fractional open area, total hole area divided by bubbling area
FPL	(m)	Flow path length
G	(kmol/(m^2·s))	Gas throughput per cross sectional bubbling area
h_f	(m)	Froth height
h_{Weir}	(mm)	Weir height
k_G	(kmol/(m^2·s))	Gas side mass transfer coefficient
k_L	(kmol/(m^2·s))	Liquid side mass transfer coefficient
L	(kmol/(m^2·s))	Liquid throughput per cross sectional bubbling area
m	(-)	Slope of the equilibrium line
N_G	(-)	Number of transfer units in gas phase
N_L	(-)	Number of transfer units in liquid phase
N_{OG}	(-)	Number of transfer units overall gas
n_{th}	(-)	Number of theoretical stages
n_{tray}	(-)	Number of actual trays

Greek

α	(-)	Relative volatility
η_{Point}	(-)	Murphree vapor phase point efficiency
η_{Tray}	(-)	Murphree vapor phase tray efficiency
$\eta_{Section}$	(-)	Section efficiency or overall efficiency
μ	(mPa·s) or (cP)	Viscosity
λ	(-)	Stripping factor = $m / (L/G)$
$\bar{\lambda}$	(-)	Stripping or absorption factor in Eq(14)

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