Continuous-time Formulation of Integrated Crude Scheduling and Planning Operations for Petrochemical Sites

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Abstract

This paper focuses on the optimization of integrated crude scheduling and refinery planning problems. Instead of using continuous time in crude scheduling and discrete time in refinery planning, a unified single-grid continuous time formulation is applied to both two parts of the problem. The Resource-Task-Network (RTN) is implemented in the crude scheduling problem to formulate the logical operational constraints. A mixed-integer linear programming (MILP) formulation is proposed. Compared to the common formulation of integration continuous time and discrete time, the proposed formulation reveals flexibility in adapting the total periods, which provides a balance to the objective and the accuracy of operations. Also, the RTN formulation avoids complicated constraints, showing a reduction in the model scale.

**Keywords**: Scheduling, Planning, Petrochemical, Continuous-time.

* 1. Introduction

Although the optimization in crude scheduling and refinery planning has been fully investigated individually over the past decades, the overall enterprise-wide optimization remains a big challenge. For the crude scheduling problem, several formulations were studied deeply, such as the State-Task-Network (STN) (Kondili et al., 1993) and the Resource-Task-Network (RTN) (Pantelides, 1994). This problem often aims to optimize the operation sequence between vessels and tanks. Thus, the time formulation plays an important role. The discrete-time simplifies the mathematical modelling while expanding the model scale. On the other hand, the continuous time could specify an accurate time point to assign the operations but more complicated logical constraints are derived. For the refinery planning problem, usually, the discrete-time formulation is implemented since the problem aims to obtain optimal planning on processing and selling over a certain time. Recently, the integrated problem of scheduling and planning has gradually drawn the attention of academics. Mouret et al (2011) investigated the integration of refinery planning and crude scheduling. A Lagrangian decomposition algorithm was applied to such large-scale problems. Further, the algorithm was used to solve the integrated scheduling and planning problem for an ethylene plant (Wang et al., 2016) and for the refinery plant (Yang et al., 2020). Continuous time and discrete time are used for scheduling and planning respectively in all these works. Even though the introduction of different time scales could capture the nature of decision-making for these two processes, the integration between continuous and discrete time formulations may add complexity to this model as extra constraints are needed to ensure the insistent. Therefore, in this paper, the unified continuous time formulation is applied to the integrated crude scheduling and refinery planning problem and a mixed-integer linear programming formulation is then obtained. The RTN is implemented to the crude scheduling to denote the logical constraints.

* 1. Problem statement

The integrated problem can be illustrated in Fig.1 shows. Usually, the crude oils are carried by marine vessels (MV) and arrive at a certain time. The crude in the vessel is then unloaded from the vessel and kept in the storage tanks (ST). The vessels may need to wait due to the availability of the dock station or STs. The crudes are stored in specific tanks. The crude oils are then transferred and blended in the following charging tank (CT) to meet the specification requirements before being processed in the crude distillation unit (CDU). The CDU is the core processing unit which separates the crudes into several streams in both crude scheduling and refinery planning. The CDU is required to operate continuously. These streams are then further processed, such as hydrotreating, cracking and reforming in the refinery units. The refinery units operate the processes at the same time as CDU. After that, final products can be obtained by blending the respective streams into corresponding pooling. The demands of all the products should be met for the entire time horizon.



Figure.1. Illustration of the integrated crude scheduling and refinery planning process.

Thus, to formulate the mathematical formulation, the following assumptions are given:

1. The time horizon for optimization is specified;
2. The vessels with arriving time, and total crude oils carried are known in advance;
3. The inventory cost, transferring cost, processing cost for each tank and operation units are given;
4. The quality specifications on the crudes and products are given;
5. The demands of final products need to be satisfied during the time horizon;
6. Fixed yield constants and linear blending are assumed for this problem.

With the assumptions above, the goal of the optimization is to minimize the total cost over the specific time horizon under the demand and capacity constraints by assigning the operations, such as unloading and transferring to the optimal time point.

* 1. Mathematical formulation

In this section, the main constraints for the continuous-time RTN crude scheduling and refinery planning mode are proposed. The constraints can be divided into time constraints, logical constraints, scheduling constraints and planning constraints.

* + 1. Time constraints

Given $n\in \{1,2,…,N\}$ continuous time slot, the start time $T\_{n}$ and duration time $ΔT\_{n}$ for each slot $n$ should be determined by Eqs. (1) and (2) over the time horizon *H.*

|  |  |
| --- | --- |
| $$T\_{1}=0$$ | (1) |
| $$T\_{n}=T\_{n-1}+ΔT\_{n-1} ∀n\in N$$ | (2) |
| $$\sum\_{n=1}^{N}ΔT\_{n}=H$$ | (3) |

* + 1. Marine vessel operation constraints

The marine vessel is the start of the crude scheduling. Given the arrival time of each vessel in the set $MV$, the crude is then unloaded to the respective ST. The constraints for the vessels are shown as follows:

|  |  |
| --- | --- |
| $$\sum\_{n=1}^{N}y\_{n,mv}^{be}=1 ∀mv\in MV$$ | (4) |
| $$\sum\_{n=1}^{N}y\_{n,mv}^{fi}=1 ∀mv\in MV$$ | (5) |
| $$R\_{n,mv}=R\_{n-1,mv}-y\_{n,mv}^{be}+y\_{n,mv}^{fi} ∀n\in N,mv\in MV$$ | (6) |
| $$F\_{n,mv}\leq F\_{mv}^{max}T\_{n}+M∙R\_{n,mv} ∀n\in N,mv\in MV$$ | (7) |
| $$F\_{n,mv}\geq F\_{mv}^{min}T\_{n}-M∙R\_{n,mv} ∀n\in N,mv\in MV$$ | (8) |
| $$\sum\_{n=1}^{N}F\_{n,mv}=I\_{mv}^{0} ∀mv\in MV$$ | (9) |

In Eqs. (4) and (5), the binary variable $y\_{n,mv}^{be}$ and $y\_{n,mv}^{fi}$ are used to denote the starting and finishing of the unloading operation for $mv$. Each vessel should start and finish unloading once during the time horizon. A binary variable $R\_{n,mv}$ is defined to represent the equipment resource. It indicates the vessel is not unloading at time slot $n$ if $R\_{n,mv}=1$. Eq. (6) shows the logical relationship of the equipment resource. In Eqs. (7) – (9), the volume of transferred crude in time slot $n $is constrained by equipment resource $R$ and total inventory $I\_{mv}^{0}$.

* + 1. Storage tank operation constraints

The STs are used to store the crudes before transferring them to the charging tanks. The constraints are shown in Eqs. (10) – (13), including the equipment resource, transferred crudes, and inventory constraints.

|  |  |
| --- | --- |
| $$R\_{n,st}=R\_{n,st}^{total}-\sum\_{mv\in MV}^{}\left(1-R\_{n,mv}\right)-y\_{n,st,ct} ∀n\in N,st\in ST,ct\in CT$$ | (10) |
| $$F\_{n,st,ct}\leq F\_{st,ct}^{max}T\_{n}+M\left(1-y\_{n,st,ct}\right) ∀n\in N,st\in ST,ct\in CT$$ | (11) |
| $$F\_{n,st,ct}\geq F\_{st,ct}^{min}T\_{n}-M\left(1-y\_{n,ct,st}\right) ∀n\in N,st\in ST,ct\in CT$$ | (12) |
| $$I\_{n,st}\leq I\_{n-1,st}+\sum\_{mv\in MV}^{}F\_{n,mv}-\sum\_{ct\in CT}^{}F\_{n,st,ct} ∀n\in N,st\in ST$$ | (13) |

Eq. (10) denotes the equipment resource constraint. By adding bounds on variable $R\_{n,st}$, the logical constraints which prevent importing and exporting materials simultaneously are imposed. Similarly, the constraints on the transferred materials are defined with Eqs. (11) – (12), while the inventory balance is shown in Eq. (13).

* + 1. Charging tank operation constraints

The charging tank $ct\in CT$ receives the crudes from ST to meet the quality requirements. The blended crudes are later transferred to CDUs for further processing. Here, the constraints of CT are simplified as follows:

|  |  |
| --- | --- |
| $$R\_{n,ct}=R\_{n,ct}^{total}-y\_{n,st,ct}-y\_{n,ct,cd} ∀n\in N,st\in ST,ct\in CT,cd\in CD$$ | (14) |
| $$F\_{n,ct,cd}\leq F\_{ct,cd}^{max}T\_{n}+M\left(1-y\_{n,ct,cd}\right) ∀n\in N,ct\in CT,cd\in CD$$ | (15) |
| $$F\_{n,ct,cd}\geq F\_{ct,cd}^{min}T\_{n}+M\left(1-y\_{n,ct,cd}\right) ∀n\in N,ct\in CT,cd\in CD$$ | (16) |
| $$I\_{n,ct}\leq I\_{n-1,ct}+\sum\_{st\in ST}^{}F\_{n,st,ct}-\sum\_{cd\in CD}^{}F\_{n,ct,cd} ∀n\in N,ct\in CT$$ | (17) |
| $$I\_{n,c,ct}=I\_{n-1,c,ct}+\sum\_{st\in ST^{c}}^{}F\_{n,c,ct}-\sum\_{cd\in CD}^{}F\_{n,c,ct,cd} ∀n\in N,c\in C,ct\in CT$$ | (18) |
| $$I\_{n,ct}=\sum\_{c\in C}^{}I\_{n,c,ct} ∀n\in N,ct\in CT$$ | (19) |
| $$\left[\begin{array}{c}y\_{n,ct,cd}\\I\_{n-1,c,ct}=I\_{n-1,ct}γ\_{c} ∀c\in C\end{array}\right] ∀n\in N,ct\in CT,cd\in CD,$$ | (20) |

Same as the ST, the constraints on CT consist of equipment resources and transferred amounts. Eq. (17) denotes the total inventory balance in the tank $ct$ while as crudes are blended in the tanks, the inventory of each crude $c\in C$ is further defined with Eq. (18) and the relationship is imposed by Eq. (19). Then the Eq. (20) represents when a certain charging tank $ct$ is transferring to a CDU, the fraction of each crude $c\in C$ should satisfy a certain composition requirement.

* + 1. Refinery planning constraints

After blending in the charging tanks, the crudes are then processed in the refinery units, starting with the CDUs. For each unit, the yields of the outlet flows are assumed as constants. Then the constraints for all the refinery units are presented as follows:

|  |  |
| --- | --- |
| $$D\_{n,cd}^{min}\leq F\_{n,cd}^{in}=\sum\_{ct\in CT}^{}F\_{n,ct,cd}\leq D\_{n,cd}^{max} ∀n\in N,cd\in CD$$ | (21) |
| $$F\_{n,u,s}^{out}=\sum\_{s'\in S^{in}}^{}F\_{n,u,s'}^{in}α\_{u,s} ∀n\in N,u\in U,s\in S^{u}$$ | (22) |
| $$C\_{u}^{min}\leq \sum\_{s^{'}\in S^{in}}^{}F\_{n,u,s^{'}}^{in}\leq C\_{u}^{max} ∀n\in N,u\in U$$ | (23) |
| $$D\_{p}^{min}\leq F\_{p}=\sum\_{n\in N}^{}\sum\_{u\in U,s\in S^{p}}^{}F\_{n,u,s}^{out}\leq D\_{p}^{max} ∀p\in P$$ | (24) |
| $$E\_{o}^{min}\sum\_{u\in U,s\in S^{p}}^{}F\_{n,u,s}^{out}\leq \sum\_{u\in U,s\in S^{p}}^{}F\_{n,u,s}^{out}E\_{o,u,s}\leq E\_{o}^{max}\sum\_{u\in U,s\in S^{p}}^{}F\_{n,u,s}^{out} $$$$∀n\in N,p\in P,o\in O$$ | (25) |

In Eq. (21), the total crudes processed in CDUs should satisfy the demand. Then for each refinery unit $u\in U$, the inlet and outlet relationship are presented with Eq. (22) while the capacity constraints are denoted with Eq. (23). For each desired product $p\in P$, the demands should be met in Eq. (24) with constraints of product property $o\in O$ shown as Eq. (25).

* + 1. Objective function

The objective function of this problem is to minimize the total cost, mainly the inventory cost and processing cost for each tank and unit, minus the income of final products. The expression of the final objective can be seen as follows:

|  |  |
| --- | --- |
| $$min z=\sum\_{n\in N,u\in U,s\in S^{u}}^{}F\_{n,u,s}^{in}β\_{u}+\sum\_{tk\in MV∪ST∪CT}^{}I\_{n,tk}β\_{tk}-\sum\_{n\in N,p\in P}^{}β\_{p}F\_{n,p}$$ | (26) |

Here, the parameter $β$ denotes the coefficients in general.

* 1. Case study

In this section, a case study is conducted with the proposed modelling framework. The integrated continuous-time scheduling and planning model (ICSP) is formulated as well as a traditional integrated continuous-time scheduling and discrete-time planning model (ICSDP) for comparison. The ICSDP model is adapted from the reference (Wang et al., 2016, 2014) which constructed the models based on the implementation of the continuous time slot in each discrete time slot. The material balance is imposed at the end of each discrete time period. All the models are built on pyomo (Bynum et al., 2021) on Windows11 using Intel(R) Xeon(R) Gold 6226R CPU @ 2.90GHz with 64GB of RAM and solved with CPLEX 22.1.1 Given the different number of time slots, the model scales and objectives are shown in Table. 1. as follows.

Table.1. Model statistics and objectives

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Continuous slots | Discrete slots | # of binary variables | # of continuous variables | # of constraints | Objective /$ |
| ICSP | 8 | - | 304 | 1,029 | 1,939 | -4,004.25 |
| 12 | 456 | 1,541 | 2,903 | -9,813.85 |
| 16 | 608 | 2,053 | 3,867 | -13,357.00 |
| 20 | 760 | 2,565 | 4,831 | -15,584.21 |
| 24 | 912 | 3,077 | 5,795 | -17,079.30 |
| ICSDP | 1 | 8 | 304 | 1,029 | 1,939 | -336.42 |
| 2 | 448 | 1,365 | 2,779 | -10,768.10 |
| 3 | 592 | 1,701 | 3,619 | -13,612.54 |

In this case study, a total of 8 days is used to represent the discrete time slots, assigning one day as a discrete single slot to refinery planning in ICSDP models. The total time slots in ICSDP would be the number of continuous time slots multiplied by the discrete time slots $N\_{con}×N\_{dis}$. While in ICSP models, the continuous time slots are also applied to the refinery planning, which offers more flexibility in choosing different time slots for the problem. For each kind of model, a better objective can be obtained when increasing the number of time slots, generating a profit from $4,004.25 to $17,079.30. However, even if the same time slots are used for both ICSP and ICSDP models, for example, 8 continuous time slots for ICSP and 1 continuous time slot for ICSDP, the ICSP model could provide a much better profit of $4,004.24 than the ICSDP at $336.42. This can be attributed to that the discrete time slots in the ICSDP model restrict the duration of time slots. While using the continuous-time formulation, the better solution is obtained with the optimal length of each time slot, even though the model scales are the same for both models. In addition to that, the ICSP usually generates a larger model than the ICSDP, such as the models of 24 time slots. This could make the problem hard to solve, especially for a large process.

* 1. Conclusion

In this paper, a unified single-grid continuous time formulation is applied to the integrated crude scheduling and refinery planning problem. To model the complex logical constraints in crude scheduling, the RTN formulation is implemented to model these constraints with equipment resources. Compared to the traditional integrated continuous-time scheduling and discrete-time planning model, the proposed formulation could provide better objectives by making flexible duration of the time slots in the planning part, revealing advantages in application although the model scale becomes larger. Future work will focus on the multiple continuous time grids for complicated integrated problems.

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