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Model of Boundary Deposition Curves for Polydisperse Suspension Sedimentation in Flow-through Systems

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The novel model of sedimentation of polydisperse suspensions have been submitted. The model is devoted to describing the process of sedimentation of a polydisperse suspension in a flow-through systems. A concept for calculating the distribution of the dispersed phase in the flow leaving the apparatus based on a new methodology of boundary deposition curves has been developed. The developed model is allowing for calculating the evolution of the sedimentation front position and the sediment surface both along the height of the vessel and along longitudinal coordinate of the through-flowing apparatuses. An expressions and computer code for calculating the evolution of the sedimentation fronts have been developed, that is of importance for calculating the kinetics of sedimentation of polydisperse suspensions in nature and industrial flow-through devices. Using the approach of boundary deposition curves, a simple and effective method for calculating changes in the fractional composition of a suspension at the outlet of a flow region has been proposed.

* 1. Introduction

Sedimentation processes are extremely widespread both in nature (Mitchell et al., 2021) and in various technological processes (Berzi and Fraccarollo, 2016). These are processes and phenomena (Wang et al., 2023) in various nature bodies of water: from rivers and lakes - to seas and oceans (Wang et al., 2024). These are also phenomena that occur in devices for the production (Chebbi, 2007) and purification (Garcia, 2007) of target technological suspensions (Schleiss et al, 2016), as well as during the purification of emissions (Hamidifar, 2024). Sedimentation processes play also an important role in pharmacology (Bürger et al., 2000) and medicine (Maki et al., 2021). From the point of view of the characteristics of these processes, first of all, it is necessary to distinguish between the characteristics of sedimentation in stationary reservoirs (Lippert and Woods, 2020,) and in flow-through systems and apparatuses (Massah et al., 2020).

Despite the widespread of sedimentation processes (Xoshimov et al, 2023), both its experimental study (Hernando et al., 2014) and theoretical description (Bürger et al., 2025) are associated with many difficulties (Kondrat’ev and Naumova, 2004), which can be explained by complex behavior of such processes in the case of multicomponent composition (Kondrat’ev and Naumova, 2007). Calculation problems become even more complex for describing the selective sedimentation of polydisperse suspensions (Yang et al., 2023) in through-flowing systems.

The fluid resistance force acting on a spherical particle (Pavlenko et al., 2021) depends on its size, relative speed of movement, viscosity of the medium and is determined by the Reynolds number (Wallwork, 2022). In conditions where such a separation of fractions by dispersion turns out to be too rough, it is necessary to consider another models (Nocoń, 2016,). In addition, this approach is acceptable only for weakly concentrated suspensions, in which there is no influence of particles of one fraction on the hydrodynamic conditions of sedimentation of another fraction (Taye, 2020). The construction of theoretical models for the sedimentation of polydisperse suspensions, even in the absence of strong interaction between particles, is a nontrivial problem and has not yet been completed (Guazzelli and Hinch, 2011). Some works consider the hindered (crowded) sedimentation of a polydisperse suspension (Lee et al., 2008). The needs of engineering practice force to look for ways to developing simplified models.

Early the authors have presented a novel approach to constructing a macroscopic sedimentation model based on the distribution function of particles in polydisperse suspension (Brener et al, 2023). The main novelty and fundamental point of the submitted in that paper model lie in that the model is constructed in the form of a diffusion equation with a source term. The results obtained can be applied to fluids with suspensions of fairly coarse particles for description the sedimentation process in stationary vessels. However, during the sedimentation of fine particles in flowing systems, the role of flow velocity and flow fluctuations on the intensity of sedimentation increases (Guazzelli and Hinch, 2011).

The main novelty and scientific contribution of the submitted here second model, unlike the first, lie in that this model is allowing for calculating the process of selective sedimentation in a through-flowing apparatuses. The submitted here model is heuristic, and it contains some hypothetical provisions that are based on the analysis of the theoretical results and the long well-known reliable experimental data (Burt, 1987), but that are not derived in detail.

Therefore, the main objective of this work is to present a new concept and methodological approach to the mathematical description of selective sedimentation in flows. The further investigation should be required on the interpretation of the control parameters of the model, as well as on verification of its adequacy on a more extensive and diverse experimental material.

* 1. Concepts Description and Theoretical Details

This section presents a general form of the model, and the model proposed is analyzed in more details for the special case of the sedimentation, which is not accompanied by the aggregation in the dispersed phase. An analysis of the submitted model in relation to the sedimentation with mutual aggregation of various fractions will be carried out and presented in subsequent publications.

* + 1. Model of boundary deposition curves in flow-through reactor. General considerations

Another approach is developed in this paper to create a sedimentation model in flow-through devices. The proposed method for calculating the selective sedimentation process in a flow-through apparatus is based on the concept of boundary sedimentation fronts for each fraction of a dispersed mixture current through the device. For each fraction, at a given longitudinal rate of the dispersion flow, two boundary fronts are formed - the upper and the lower.

The upper front is a curve describing the trajectory of a particle of a given fraction entering the working volume of the flow-through apparatus at the upper point of the initial section. If, at a given flow rate, particles of a given fraction do not reach the final outlet section of the apparatus, then the concept of a lower front does not make sense, since the upper front ends at the bottom of the apparatus.

If the upper front reaches the outlet section of the apparatus, then the lower front is a curve describing the trajectory of a particle of a given fraction entering the working volume of the flow-through apparatus at a certain point of the initial section and reaching the outlet section at the lower point, i.e. at the bottom level.

The upper and lower boundary deposition curves for each fraction limit the breakthrough band of the given fraction through the flow apparatus. Then, under the uniform distribution of different fractions in the initial section at the entrance to the apparatus, the change in the share of any fraction in the dispersed composition at the exit from the apparatus can be estimated by the ratio of the width of the breakthrough band to the total exit cross-section.

The new model is based on the following assumptions:

First, it is possible to clearly identify a certain finite number of fractions that differ in order, i.e. in size.

Secondly, it is assumed that the composition of the dispersed phase is homogeneous, i.e. all particles of the solid phase carried away by the flow consist of one substance.

Thirdly, the partial concentration of the solid phase is not too high. Then, the mutual influence of different fractions on the intensity of their sedimentation can be neglected. Note that this assumption is not critical, and the model built further can be developed without this assumption. However, in this work, such an assumption is made to facilitate a clear description of the structure of the novel model.

Fourth, it is assumed that the shape coefficients of different particles do not differ significantly. Thus, it is possible to accept a uniform dependence of both the coefficient of particle entrainment by the flow and the parameters that determine the settling rate of particles on the order of particles, i.e. on their characteristic size.

Fifthly, the distribution of particles of various fractions at the entrance to the apparatus is assumed to be uniform over the entire inlet cross-section. Thus, the partial distribution function in each local region of the input section is the same.

From the five main premises highlighted above, in turn, the provisions of the concept for constructing a model follow: Its own deposition front can be constructed for each fraction. This follows from first, third and fifth assumptions. The second and fourth assumptions make it possible to use uniform calculated dependencies for calculating deposition fronts of various fractions.

* + 1. Detail problem statement and model concept

1. In the stationary mode, there are formed certain curves, which can be denoted as fraction clarification or sedimentation fronts.

2. There is a fraction whose extreme clarification front extends from the top point of the initial cross-section to the lower point of the final cross-section of the through-flowing apparatus. This front denotes the finest fraction of those particles that are completely deposited along the apparatus and do not fall into the suspension flow leaving the apparatus . The plot *C3* is shown in Figure 1.

3. The clarification fronts of finest fractions do not end at the lowest point of the apparatus. Therefore, particles of such fractions leave the apparatus together with a flowing stream.

4. Let the symbol  denotes the front of clarification of the critical fraction (i.e., the finest of the fractions that are completely deposited in the volume of the apparatus and are not represented in the flow leaving the apparatus).

5. Thus, the breakthrough of a fraction of order  into the flow leaving the apparatus occurs in the “band” between the two boundary deposition curves (Figure 2): and . The lower boundary curve  for a given fraction represents the trajectory of a particle of this fraction, entering the working volume of the apparatus at a certain point below the upper left point and exiting the apparatus at the lower right point of the outlet section.

6. Let the symbol  denotes the front of clarification of the finest from all fractions in the mixture. Then the average concentration of any fraction from the interval  at the outlet of the apparatus for the case of a discrete distribution of fractions uniform over the inlet section can be described by the formula

. (1)

7. If, in addition, the rate of sedimentation of the fractions weakly depends on the distance to the lower wall (bottom) of the apparatus, and it depends on the fraction order only, then the sedimentation fronts will be straight lines. Then the previous formula can be rewritten as

. (2)



Figure 1: Typical plots for the boundary deposition curves



Figure 2: Typical plots for the top and bottom deposition curves applying to given particle order

This assumption can be more or less acceptable for low-concentration dispersions.

* + 1. Brief results of numerical simulation

The main fragment of the code used for the numerical investigations with allowing for the flow random fluctuations depicts in Figure 3.



Figure 3: Main fragment of computational code

Below the main previous results of an analysis applying to the model of sedimentation into the through-flowing apparatuses have been submitted (Figures 4, 5).



*A* *B*

*A*- Conventional horizontal entraining velocity Ug=2; *B*- Conventional horizontal entraining velocity Ug=6.

Figure 4: *Typical boundary deposition curves in through-flowing systems for different conditional velocities*



*A* *B*

1. Conventional particle order r=1 for conventional entraining velocity Ug=6; *B*- Various conventional particles orders for horizontal entraining velocity Ug=8

Figure 5: *Typical boundary deposition curves in through-flowing systems for different particles orders*

* 1. Conclusions

The submitted model that based on the novel concept of boundary sedimentation curves makes it possible to calculate all the important for practice dynamical characteristics of the sedimentation of polydisperse suspensions for selective sedimentation in through-flowing systems. The analysis of the obtained dependences and the type of graphs is in good agreement with the known studied patterns of sedimentation. A fairly simple and effective program code has been developed for a fundamentally new model of deposition in a flow system. Similar accelerated calculations using this model can be carried out with different initial data. For reliable practical application, this model requires the identification of control parameters for specific physicochemical systems. Only after that it will be possible to recommend a reliable step-by-step algorithm for using the model in practice.

Nomenclature

 – lower boundary sedimentation curve – fraction with number order

 – upper boundary sedimentation curve

– liquid flow rate, m/s

 – conditional lower (0) and upper (f) curves locations

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